

TOPPERS VILLAGE



Class 11 Mathematics

Chapter 16: Probability

Class Notes (pdf)

YouTube पर इस Chapter के FREE Videos देखिए ... (Playlist की Link नीचे है) ↓↓↓

<https://www.youtube.com/watch?v=k7BNrGn9G70&list=PLPFrnOppwwkSh7n8wQAt188pz3aaJF>

EzB

- Axiomatic Approach of Probability

- Probability = $\frac{\text{no. of favourable outcomes}}{\text{total no. of outcomes}}$
(of an event E)
 $= P(E)$

$$\boxed{0 \leq P(E) \leq 1}$$

- Outcomes = Results.

- Random Experiment: an experiment which has two or more well defined outcomes, but we can not predict the next outcome in advance.

e.g. Tossing a fair coin
Throwing a fair Die

- Sample Space: set of all possible outcomes in a random experiment.

e.g. Random Experiment $\rightarrow$ Tossing a Die.
Sample Space (S) $\rightarrow S = \{1, 2, 3, 4, 5, 6\}$

- Event: what we want
(E) (subset of sample space)

e.g. Random Experiment $\rightarrow$ Tossing a Die
Sample Space $S = \{1, 2, 3, 4, 5, 6\}$

We want PRIME NUMBERS

Event (E) $E = \{2, 3, 5\}$

$$P(\text{Prime}) = P(E) = \frac{3 \leftarrow \{2, 3, 5\}}{6 \leftarrow \{1, 2, 3, 4, 5, 6\}} \\ = \frac{1}{2} = 0.5$$

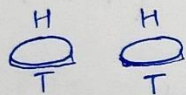
Some Important Sample Spaces

- Tossing a fair Coin 

$S = \{H, T\}$ = Sample Space

no. of elements in $S = n(S) = 2 = 2^1$

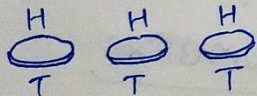
- Tossing 2 coins / (Tossing a Coin Twice)



$S = \{HH, HT, TH, TT\}$

$n(S) = 4 = 2^2$

- Tossing 3 Coins / (Tossing a Coin Thrice)



$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$n(S) = 8 = 2^3$

- Roll a Die (Throw a Die)



$S = \{1, 2, 3, 4, 5, 6\}$

$n(S) = 6$

- Roll 2 Dice (Roll a Die twice)

$S = \left\{ \begin{array}{l} (1,1) (1,2) (1,3) (1,4) (1,5) (1,6) \\ (2,1) (2,2) (2,3) (2,4) (2,5) (2,6) \\ (3,1) (3,2) (3,3) (3,4) (3,5) (3,6) \\ (4,1) (4,2) (4,3) (4,4) (4,5) (4,6) \\ (5,1) (5,2) (5,3) (5,4) (5,5) (5,6) \\ (6,1) (6,2) (6,3) (6,4) (6,5) (6,6) \end{array} \right\}$



1
2
3
4
5
6



1
2
3
4
5
6

$n(S) = 36$

- Toss a Coin & Roll a Die



$S = \left\{ \begin{array}{l} (H,1) (H,2) (H,3) (H,4) (H,5) (H,6) \\ (T,1) (T,2) (T,3) (T,4) (T,5) (T,6) \end{array} \right\}$

$n(S) = 2 \times 6 = 12$

1
2
3
4
5
6

A Well shuffled Deck of 52-Playing Cards

Total = 52 Cards

Black Cards = 26

Red Cards = 26



Colour = 2

SPADE = 13



CLUB = 13



DIAMOND = 13



HEART = 13



Suit = 4

Face Cards (=12)	Ace	A	A	A	Honour Cards (=16)
	King	K	K	K	
	Queen	Q	Q	Q	
	Jack	J	J	J	
	10	10	10	10	
	9	9	9	9	Number Cards (=36)
	8	8	8	8	
	7	7	7	7	
	6	6	6	6	
	5	5	5	5	
	4	4	4	4	
	3	3	3	3	
	2	2	2	2	

Some other important tools
used in probability.

$$\star n_{C_r} = \frac{n!}{(n-r)! r!} = \text{no. of ways to } \underline{\text{select}} \text{ } r \text{ objects} \\ \text{out of } n\text{-different objects.}$$

$$\star n_{P_r} = \frac{n!}{(n-r)!} = \text{no. of ways to } \underline{\text{arrange}} \text{ } r\text{-objects} \\ \text{out of } n\text{-different objects}$$

e.g. R. Experiment: Draw 1 card from 52 playing Cards

$$n(S) = 52C_1 = 52 \\ \uparrow \\ \text{(Sample Space)}$$

e.g. A. Experiment: Draw 2 cards from 52 playing Cards

$$n(S) = 52C_2 = \frac{52!}{50! \times 2!} = \frac{52 \times 51 \times \cancel{50!}}{\cancel{50!} \times 2 \times 1} = 26 \times 51 \\ \uparrow \\ \text{Sample Space}$$

Toppers Village Class-11-Maths.

Chapter-16

Probability

Exercise 16.1

Sample Space : "Set of all possible outcomes"

Q.1 A coin $\rightarrow$ 3 times toss $\begin{matrix} H \\ \circ \\ T \end{matrix} \begin{matrix} H \\ \circ \\ T \end{matrix} \begin{matrix} H \\ \circ \\ T \end{matrix}$

Sample space = $S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

Q.2



1st time

1
2
3
4
5
6



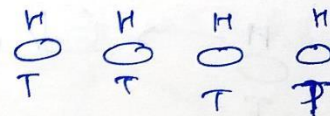
2nd time

1
2
3
4
5
6

$S = \left\{ \begin{matrix} (1,1) & (1,2) & \dots & (1,6) \\ (2,1) & (2,2) & \dots & (2,6) \\ \vdots & \vdots & & \vdots \\ (6,1) & (6,2) & \dots & (6,6) \end{matrix} \right\}$

$n(S) = 36$

Q.3



$S = \{HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTHT, THTT, HTTH, THTH, TTHH, HTTT, THTT, TTHT, TTTT, TTTT\}$

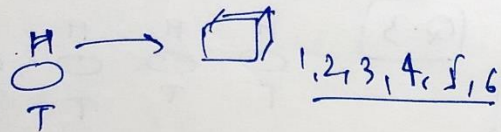
Q.4



$\frac{1}{2}$
3
4
5
6

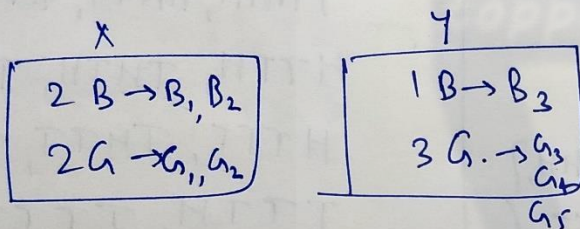
$S = \{H_1, H_2, H_3, \dots, H_6, T_1, T_2, T_3, \dots, T_6\}$

Q.5



$$S = \{ T, H_1, H_2, H_3, H_4, H_5, H_6 \}$$

Q.6

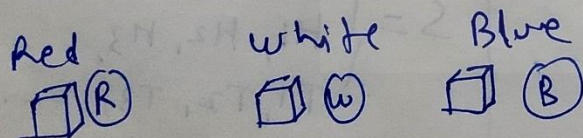


I $\rightarrow$ Room $\checkmark$

II $\rightarrow$ Person. $\checkmark$

$$S = \{ XB_1, XB_2, XG_1, XG_2, YB_3, YG_3, YG_4, YG_5 \}$$

Q.7



$$S = \{ R_1, R_2, R_3, \dots, R_6, W_1, W_2, W_3, \dots, W_6, B_1, B_2, B_3, \dots, B_6 \}$$

Q.8

(i) $B \rightarrow$ Boy

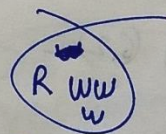
$G \rightarrow$ Girl

$$S = \{ \underline{BB}, \underline{BG}, \underline{GB}, \underline{GG} \}$$

(ii) $S = \{ 0, 1, 2 \}$

number of Girls.
0, 1, 2, 3, ...

Q.9

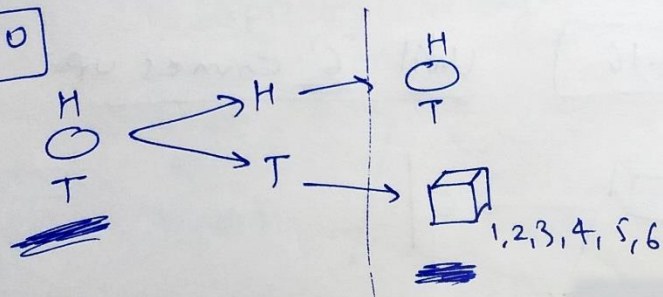


Bag

(without replacement)

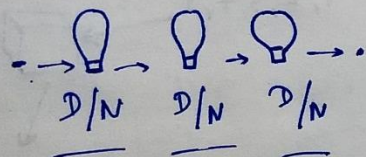
$$S = \{ RW, WR, ww \}$$

Q.10



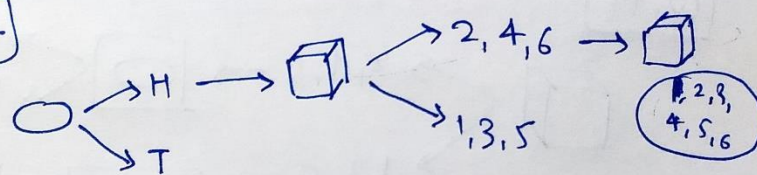
$$S = \{ HH, HT, TH, TT, T1, T2, T3, T4, T5, T6 \}$$

Q.11



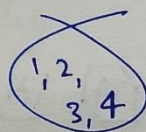
$$S = \{ DDD, DDN, DND, NDD, DNN, NDN, NND, NNN \}$$

Q.12



$$S = \{ T, H1, H3, H5, (H2, 1), (H2, 2), \dots, (H2, 6), (H4, 1), (H4, 2), \dots, (H4, 6), (H6, 1), (H6, 2), \dots, (H6, 6) \}$$

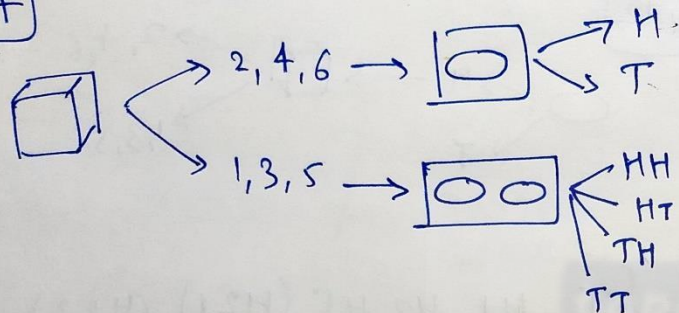
Q.13



Without replacement

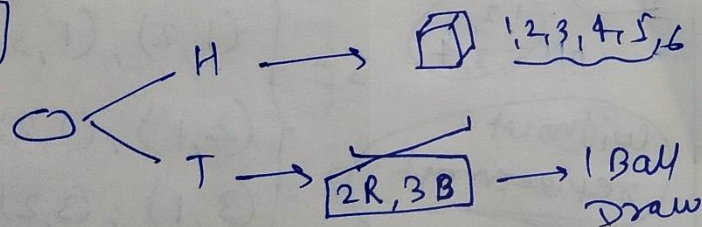
$$S = \{ (1, 2), (1, 3), (1, 4), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (3, 4), (4, 1), (4, 2), (4, 3) \}$$

Q.14



$$S = \left\{ \begin{array}{l} 2H, 2T, 4H, 4T, 6H, 6T, \\ 1HH, 1HT, 1TH, 1TT \\ 3HH, 3HT, 3TH, 3TT \\ 5HH, 5HT, 5TH, 5TT \end{array} \right\}$$

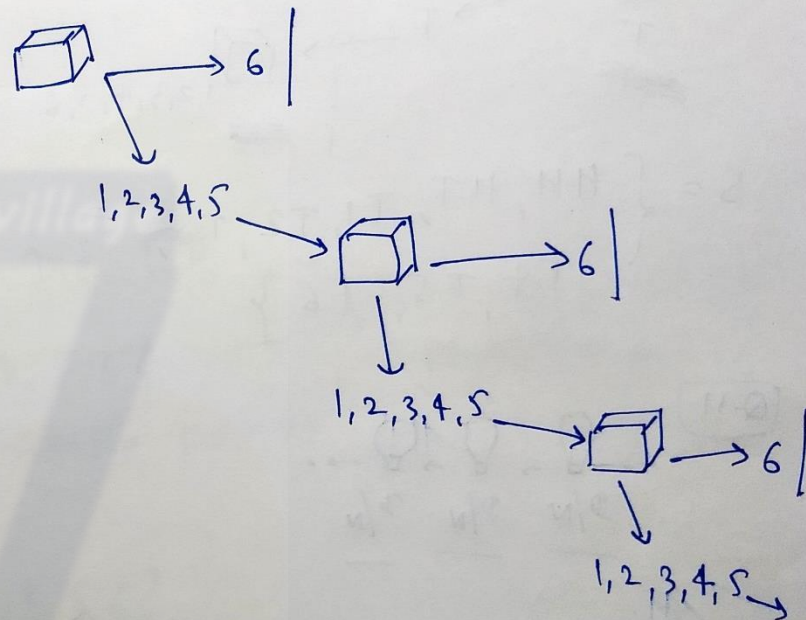
Q.15



$$S = \left\{ \begin{array}{l} H1, H2, H3, H4, H5, H6, \\ TR_1, TR_2, TB_1, TB_2, TB_3 \end{array} \right\}$$

Q.16

until '6' comes up.



$$S = \left\{ \begin{array}{l} 6, \\ (1, 6), (2, 6), (3, 6), (4, 6), (5, 6), \\ (1, 1, 6), (1, 2, 6), (1, 3, 6) \dots (1, 5, 6), \\ (2, 1, 6), (2, 2, 6), (2, 3, 6) \dots (2, 5, 6) \\ \dots \end{array} \right\}$$

Event: (E) 'what we want' / subset of sample space (S)

e.g. Random Experiment: Rolling a Die 
Sample Space = $S = \{ \underbrace{1, 2, 3, 4, 5, 6}_{\text{sample points}} \}$ $\rightarrow$ (No. of Sample points) = 6

Event: we want prime No.

$$E = \{2, 3, 5\} \subset \{1, 2, 3, 4, 5, 6\}$$

$$\boxed{E \subseteq S}$$

$$P(E) = P(\text{Prime}) = \frac{3}{6} = \frac{1}{2}$$

Note: $0 \leq P(E) \leq 1$

$P(E) = 1 \Rightarrow \text{chance} = 100\% \Rightarrow E = \text{Sure Event}$

$P(F) = 0 \Rightarrow \text{chance} = 0\% \Rightarrow F = \text{Impossible Event}$

Different Types of Events:

Probability $\leftrightarrow$ Set

① Impossible Event: $P(E) = 0 = P(\phi)$ $\phi = \text{empty set} = \{ \}$

② Sure Event: $P(E) = 1$

③ Simple Event: Event which has ^{only} one sample point.

e.g. Tossing a coin. $\rightarrow S = \{H, T\}$

Event 'E' = getting Head

Simple event. $\rightarrow E = \{H\}$

④ Compound Event

(more than one sample point)

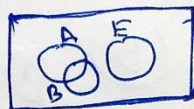
e.g. Experiment $\rightarrow$ Rolling a Die
 $S = \{1, 2, 3, 4, 5, 6\}$

Event $E = \text{Even no.} = \{2, 4, 6\}$

Start to
~~link~~
Set $\rightarrow$ Probability

Algebra of Events.

Event $\rightarrow$ set



$U = \text{Sample Space} = S$

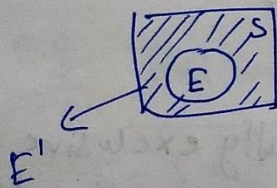
① Complementary Event

Event $\rightarrow E$

Complementary Event $= E' = \bar{E} = E^c$

$\rightarrow$ not 'E'

$$E' = S - E$$



e.g. Roll a Die 

$$S = \{1, 2, 3, 4, 5, 6\}$$

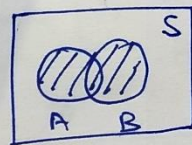
$E =$ no. less than 3

$$E = \{1, 2\}$$

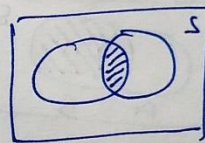
$$E' = \{3, 4, 5, 6\}$$

② Event 'A' or 'B' $= A \cup B$

③ Event 'A' and 'B' $= A \cap B$



$A \cup B$



$A \cap B$

(Common Part)

e.g. Rolling a Die 

$$S = \{1, 2, 3, 4, 5, 6\}$$

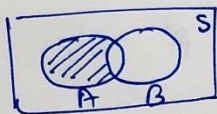
$E =$ no. less than '3' $= \{1, 2\}$

$F =$ odd no. $= \{1, 3, 5\}$

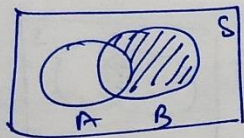
$$\left. \begin{array}{l} E \cup F \\ \{1, 2, 3, 5\} \\ P(E \cup F) = \frac{4}{6} \end{array} \right\} \begin{array}{l} E \cap F \\ \{1\} \\ P(E \cap F) = \frac{1}{6} \end{array}$$

④ Event 'A but not B'

$$A - B = A \cap B'$$



Event 'B but not A' = $B - A$
 $= B \cap A'$



e.g. Roll a Die 
 $S = \{1, 2, 3, 4, 5, 6\}$

$E = \text{No. less than 3} = \{1, 2\}$

$F = \text{odd no.} = \{1, 3, 5\}$

↳ Find prob. of getting
 odd no. but not less
 than '3'.

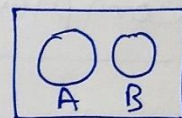
$$\Rightarrow F - E = \{1, 3, 5\} - \{1, 2\} \\ = \{3, 5\}$$

$$P(F - E) = \frac{2}{6}$$

⑤ Mutually Exclusive Events.

which have
 nothing in
 common.

Disjoint Sets



$$\underline{A \cap B} = \{ \} = \phi$$

e.g. Rolling a Die 

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$E = \{1, 2\} \quad H = \{6\}$$

$$F = \{1, 3, 5\}$$

$E \ \& \ F \rightarrow \text{not mutually exclusive Events.}$

$E \ \& \ H \rightarrow \text{MEE} \checkmark$

$F \ \& \ H \rightarrow \text{MEE} \checkmark$

⑥ Exhaustive Events:

$E_1, E_2, \dots, E_n$ are said to be Exhaustive Events if at least one of them necessarily occurs whenever ~~is~~ experiment is performed.

mathematically.

$$E_1 \cup E_2 \cup E_3 \cup \dots \cup E_n = \overset{\substack{\text{Sample} \\ \text{space}}}{S}$$

e.g. Rolling a Die. 

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$E = \{1, 3, 5\} \quad A = \{2, 3\}$$

$$F = \{2, 4, 6\} \quad B = \{1, 4, 5, 6\}$$

$$C = \{5\}$$

⑦ Mutually Exclusive & Exhaustive Events:

$$\underline{E \& F} \rightarrow \begin{cases} E \cap F = \phi \\ E \cup F = S \end{cases}$$

$$\underline{A \& B} \rightarrow \begin{cases} A \cap B = \phi \\ A \cup B = S \end{cases}$$

✓ $\underline{E \& F} \rightarrow E \cup F = \{1, 2, 3, 4, 5, 6\} = S$

✓ $\underline{A \& B} \rightarrow A \cup B = \{1, 2, 3, 4, 5, 6\} = S$

✗ $\underline{E \& A} \rightarrow E \cup A = \{1, 2, 3, 5\} \neq S$

✗ $\underline{E, F, A} \rightarrow E \cup F \cup A = \{1, 2, 3, 4, 5, 6\} = S$

Toppers Village Class - II Maths

Exercise. 16.2

Probability

Sure Event

$A \cup B$ 'A or B'

Impossible Event

$A \cap B$ 'A & B'

Simple Event

$A - B$ 'A but not B'

Compound Event

Mutually Exclusive Events.

Complementary Event

Exhaustive Events.

Q.1



$$S = \{1, 2, 3, 4, 5, 6\}$$

$$E = \{4\}$$

$$F = \{2, 4, 6\}$$

Empty
 $\{\}$

Mutually Exclusive Events $\therefore E \cap F = \phi$

$$E \cap F = \{\underline{4}\} \cap \{2, \underline{4}, 6\} = \{4\} \neq \phi$$

Common

E & F are not MEE

Q.2



Die

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$A = \{1, 2, 3, 4, 5, 6\}$$

$$B = \{\} = \phi$$

$$C = \{3, 6\}$$

$$D = \{1, 2, 3\}$$

$$E = \{6\}$$

$$F = \{3, 4, 5, 6\}, \quad \overset{\text{not } F}{\uparrow} \quad F' = \{1, 2\}$$

$$A \cup B = \{1, 2, 3, 4, 5, 6\}$$

$$A \cap B = \phi = \{\}$$

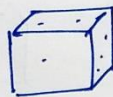
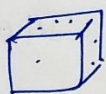
$$A - C = \{1, 2, 3, 4, 5, 6\} - \{3, 6\}$$

$$= \{1, 2, 4, 5\}$$

$$E \cap F' = \{6\} \cap \{1, 2\}$$

$$= \phi = \{\}$$

Q.3



$$S = \left\{ \begin{array}{ccc} (1,1) & (1,2) & \dots & (1,6) \\ (2,1) & (2,2) & \dots & (2,6) \\ \vdots & \vdots & & \vdots \\ (6,1) & (6,2) & \dots & (6,6) \end{array} \right\}$$

A: Sum is greater than 8



9, 10, 11, 12

$$A = \left\{ \begin{array}{l} (6,3), (5,4), (4,5), (3,6) \\ (6,4), (5,5), (4,6), \\ (6,5), (5,6) \\ (6,6) \end{array} \right\}$$

B: 2 occurs on either Die.

$$B = \left\{ \begin{array}{l} (2,1), (2,2), (2,3), (2,4), (2,5), (2,6) \\ (1,2), (3,2), (4,2), (5,2), (6,2) \end{array} \right\}$$

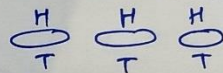
~~Q.4~~

C: Sum at least 7
& multiple of 3.

Sum $\rightarrow 7, 8, 9, 10, 11, 12$

$$C = \left\{ \begin{array}{l} (6,3), (5,4), (4,5), (3,6) \\ (6,6) \end{array} \right\}$$

Q.4



$$S = \left\{ \begin{array}{l} HHH, HHT, HTH, THH, HTT, \\ THT, TTH, TTT \end{array} \right\}$$

$$A: 3 \text{ H's} = \{ \underline{HHH} \}$$

$$B: 2 \text{ H's \& 1 T} = \{ HHT, HTH, THH \}$$

$$C: 3 \text{ T's} = \{ \underline{TTT} \}$$

$$D: \text{H on the first coin} = \left\{ \begin{array}{l} HHH, HHT, \\ HTH, HTT \end{array} \right\}$$

(i) Mutually Exclusive Events $E \cap F = \phi$

Disjoint

$$\frac{A \& B}{\underline{A \cap B = \phi}} \mid \frac{A \& C}{\underline{A \cap C}} \mid \frac{B \& C}{\underline{B \cap C}} \mid \frac{C \& D}{\underline{C \cap D}}$$

(ii) Simple events.

↓
A, C

only one element

(iii) Compound Events.

B, D

more than one ~~less~~ element

Q.6



$$S = \left\{ \begin{array}{cccc} (1,1) & (1,2) & \dots & (1,6) \\ (2,1) & (2,2) & \dots & (2,6) \\ \vdots & \vdots & & \vdots \\ (6,1) & (6,2) & \dots & (6,6) \end{array} \right\}$$

A = even no. of first die

B = odd no. of first die

C = Sum $\leq 5 \rightarrow$ Sum $\rightarrow \begin{matrix} 5 \\ 4 \\ 3 \\ 2 \end{matrix}$

$$A = \left\{ \begin{array}{l} (2,1), (2,2), \dots, (2,6) \\ (4,1), (4,2), \dots, (4,6) \\ (6,1), (6,2), \dots, (6,6) \end{array} \right\}$$

$$B = \left\{ \begin{array}{l} (1,1), (1,2), \dots, (1,6) \\ (3,1), (3,2), \dots, (3,6) \\ (5,1), (5,2), \dots, (5,6) \end{array} \right\}$$

$$C = \left\{ \begin{array}{l} (4,1), (3,2), (2,3), (1,4) \\ (3,1), (2,2), (1,3), (1,2), (2,1), (1,1) \end{array} \right\}$$

Q.5.

self

Q. 7.

(i) A & B are mutually exclusive events. (True)

(ii) A & B are mutually exclusive & exhaustive events.

$$\begin{array}{ccc} \downarrow & & \downarrow \\ A \cap B = \emptyset & & A \cup B = S \\ \checkmark & & \downarrow \quad \downarrow \\ & & \begin{pmatrix} \text{rows} \\ 1, 3, 5 \end{pmatrix} \quad \begin{pmatrix} \text{rows} \\ 2, 4, 6 \end{pmatrix} \end{array}$$

(True)

(iii) $A = B'$ (True)

(iv) A & C $\rightarrow$ mutually exclusive
 $A \cap C \neq \emptyset$ (False)

(v) A & B' are mutually exclusive. (False)

$$\begin{array}{l} A \cap (B') \neq \emptyset \\ A \cap A \neq \emptyset \\ A \neq \emptyset \end{array} \quad \text{By (iii) Part}$$

(vi) A', B', C are mutually exclusive & ~~exhaustive~~ Exhaustive

$$\begin{array}{l} \text{Red circle: } \cancel{A' \cap B' \cap C = \emptyset} \\ \Rightarrow \cancel{B \cap A \cap C = \emptyset} \\ \text{only for 2} \\ A' \cap B' = B \cap A = \emptyset \\ \underline{A'} \cap \underline{C} = B \cap C \neq \emptyset \quad \text{Not MEE} \\ B' \cap C \end{array}$$

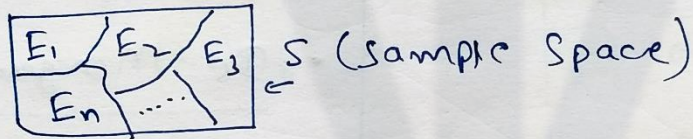
$$\begin{array}{l} A = B' \\ A' = (B')' \\ A' = B \end{array}$$

Probability : Axiomatic Approach to Probability

- Probability of the Event (E)

$$= P(E) = \frac{\text{no. of outcomes favourable to E}}{\text{total possible outcomes}}$$

- Conditions:



$$(i) \quad 0 \leq P(E) \leq 1$$

$$(ii) \quad P(E_1) + P(E_2) + P(E_3) + \dots + P(E_n) = P(S) = 1$$

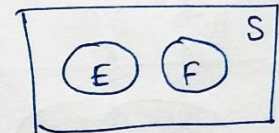
$$S = \{a_1, a_2, a_3, \dots, a_n\}$$

$$E_1 = \{a_1\} \quad \text{Simple Event}$$

$$E_2 = \{a_2\} \quad \text{simple —}$$

$$E_n = \{a_n\}$$

- Mutually Exclusive Events: E & F

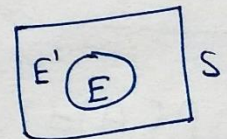


$$E \cap F = \phi = \{ \}$$

$$P(E \cap F) = 0$$

$$P(\text{both E \& F}) = 0$$

- Complementary Events.



$$E \cup E' = S$$

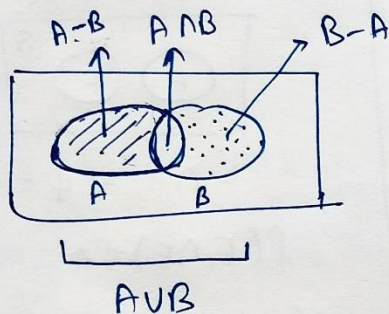
$$P(E \cup E') = P(S) = 1$$

$$\Rightarrow P(E) + P(E') = 1$$

$$P(E') = 1 - P(E)$$



$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



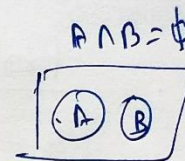
$$P(A \cup B) = P(A - B) + P(A \cap B) + P(B - A)$$

$$P(A) = \underline{P(A - B)} + P(A \cap B)$$

$$P(A \cap B) = P(A - B) = P(A) - P(A \cap B)$$

$$\underline{P(A' \cap B) = P(B - A) = P(B) - P(A \cap B)}$$

- If A & B are mutually exclusive events then:



$$P(A \cup B) = P(A) + P(B) - \underline{\underline{P(A \cap B)}}$$

$$\boxed{P(A \cup B) = P(A) + P(B)}$$

0

- De Morgan's Law:

$$\Rightarrow P(\underline{(A \cup B)'}) = P(A' \cap B')$$

$1 - P(A \cup B)$

↑ Question

$$\Rightarrow P(\underline{(A \cap B)'}) = P(A' \cup B')$$

$1 - P(A \cap B)$

Toppers Village Class - 11 - Maths

Exercise - 16.3 Probability

Q.1 Sample Space = $S = \{w_1, w_2, w_3, w_4, w_5, w_6, w_7\}$

(a) $0 \leq P(w_i) \leq 1$ ✓

$P(w_1) + P(w_2) + \dots + P(w_7)$

$\sum_{i=1}^7 P(w_i) = 0.1 + 0.01 + 0.05 + 0.03 + 0.01 + 0.2 + 0.6$

$= \underline{\underline{1.00}}$

$\sum P(w_i) = 1$ Valid

(b) $\frac{1}{7} \quad \frac{1}{7} \quad \frac{1}{7} \quad \frac{1}{7} \quad \frac{1}{7} \quad \frac{1}{7} \quad \frac{1}{7}$

~~$P(w_i)$~~ $0 \leq P(w_i) \leq 1$
 $\left(\frac{1}{7}\right)$ ✓

Valid.

$\sum_{i=1}^7 P(w_i) = 7 \times \frac{1}{7} = 1$ ✓

(c) 0.1 0.2 0.3 0.4 0.5 0.6 0.

$0 \leq P(w_i) \leq 1$ ✓

$\sum_{i=1}^7 P(w_i) = 2.8 \neq 1$ X
not valid

(d) -0.1 0.2 0.3 0.4 -0.2 0.1 0.3

↑

Negative

$P(w_i) = -0.1$ Not possible.

Not valid.

③ $\frac{1}{14} \quad \frac{2}{14} \quad \frac{3}{14} \quad \frac{4}{14} \quad \frac{5}{14} \quad \frac{6}{14} \quad \frac{15}{14}$
 $\uparrow$

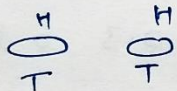
$\frac{15}{14} > 1$

$P(\omega_7) > 1$

$0 \leq P(\omega_i) \leq 1$

Not valid.

[Q.2]



$S = \{HH, HT, TH, TT\}$

$P(\text{at least one tail})$

$= \frac{3}{4} \leftarrow HT, TH, TT$

[Q.3] a die

$S = \{1, 2, 3, 4, 5, 6\} \leftarrow \text{total}$

(i) $P(\text{Prime no.}) = \frac{3 \leftarrow \overbrace{2, 3, 5}^{\text{prime}}}{6 \leftarrow \text{total}}$
 $= \frac{1}{2}$

(ii) $P(\text{number} \geq 3) = \frac{4 \leftarrow \overbrace{3, 4, 5, 6}^{\geq 3}}{6 \leftarrow \text{total}}$
 $= \frac{2}{3}$

(iii) $P(\text{no.} \leq 1) = \frac{1 \leftarrow \{1\}}{6}$

(iv) $P(\text{no.} > 6) = \frac{0 \leftarrow \text{no. } \neq 6}{6}$
 $= 0$

impossible event

(v) $P(\text{no.} < 6) = \frac{5 \leftarrow \{1, 2, 3, 4, 5\}}{6}$

Q.4 A card is selected from 52 cards.

any one of the 52 cards can come here.

(i) Sample space = Set of all cards.

$$n(S) = 52$$

(ii) $P(\text{ace of spade}) = \frac{1}{52} \leftarrow \text{total}$

(iii) $P(\text{ace}) = \frac{4}{52} = \frac{1}{13}$

$$P(\text{Black card}) = \frac{26}{52} = \frac{1}{2}$$

Q.5 Coin Die



1, 2, 3, 4, 5, 6

Value₁

+

Value₂

(i) Sum = 3

$$S = \left\{ (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \right\}$$

$$P(\text{Sum} = 3) = \frac{1 \leftarrow (1,2)}{12 \leftarrow \text{total}}$$

(ii) Sum = 12

$$P(\text{Sum} = 12) = \frac{1 \leftarrow (6,6)}{12 \leftarrow \text{total}}$$

Toppers Village Class-11-Maths

Exercise-16.3 Probability

Q.6 4 Men, 6 Women
1 person

$$P(\text{woman}) = \frac{6 \leftarrow \text{women}}{10 \leftarrow \text{total}} \\ = \frac{3}{5}$$

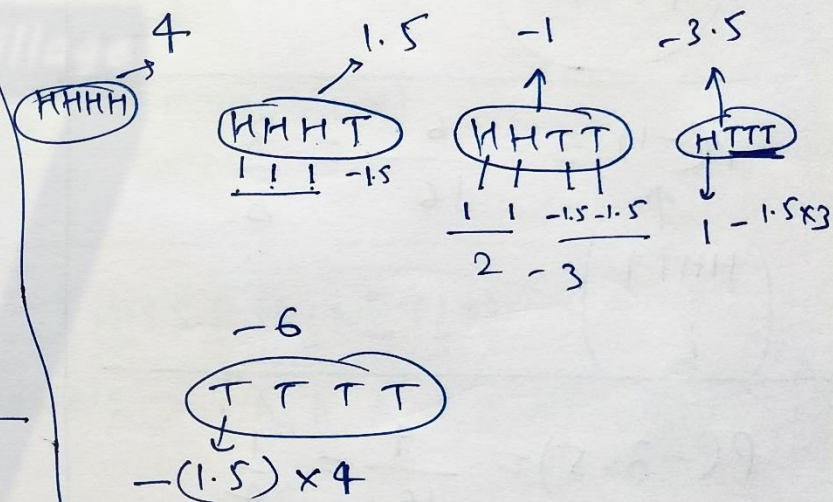
Q.7 $\begin{matrix} H \\ \bigcirc \\ T \end{matrix} \begin{matrix} H \\ \bigcirc \\ F \end{matrix} \begin{matrix} H \\ \bigcirc \\ T \end{matrix} \begin{matrix} H \\ \bigcirc \\ T \end{matrix}$

$$S = \left\{ \begin{array}{l} \text{HHHH, HHHH, HHTH, HTHH, THHH,} \\ \text{HHTT, HTHT, THTT, HTTH, THTH,} \\ \text{TTTH, TTTT, THTT, TTHT, TTTH,} \\ \text{TTTT} \end{array} \right\}$$

$$H \rightarrow +1 \text{ ₹}$$

$$T \rightarrow -1.5 \text{ ₹}$$

Different types of amounts



Different amounts

$$= \{4, 1.5, -1, -3.5, -6\}$$

$$P(\text{amt} = 4) = \frac{1 \leftarrow \text{HHHH}}{16}$$

$$P(1.5) = \frac{4}{16} = \frac{1}{4}$$

$\begin{pmatrix} \text{HHHT} \\ \text{HHTH} \\ \text{HTHH} \\ \text{T HHH} \end{pmatrix}$

$$P(-1) = \frac{6}{16} = \frac{3}{8}$$

$\begin{pmatrix} \text{HHTT} \\ \vdots \end{pmatrix}$

$$P(-3.5) = \frac{4}{16} = \frac{1}{4}$$

$\begin{pmatrix} \text{HTTT}, \dots \end{pmatrix}$

$$P(-6) = \frac{1}{16}$$

$\begin{pmatrix} \text{TTTT} \end{pmatrix}$

Q.8

$\begin{matrix} \text{H} & \text{H} & \text{H} \\ \circ & \circ & \circ \\ \text{T} & \text{T} & \text{T} \end{matrix}$

Sample Space = $S = \left\{ \begin{matrix} \text{HHH}, \text{HHT}, \text{HTH}, \text{THH}, \\ \text{HTT}, \text{THT}, \text{TTH}, \text{TTT} \end{matrix} \right\}$
 $n(S) = 8$

(i) $P(3H) = \frac{1}{8}$ ← HHH

(ii) $P(2H) = \frac{3}{8}$

(iii) $P(\text{at least } 2H) = \frac{3 + 1}{8} = \frac{4}{8} = \frac{1}{2}$

$\begin{matrix} \swarrow 2H & \swarrow 3H \\ 3 & 1 \end{matrix}$

$\begin{matrix} \downarrow \\ 2H \text{ or } 3H \end{matrix}$

(iv) $P(\text{at most } 2H) = \frac{1 + 3 + 3}{8} = \frac{7}{8}$

$\begin{matrix} \swarrow 0H & \swarrow 1H & \swarrow 2H \\ 1 & 3 & 3 \end{matrix}$

$\begin{matrix} \swarrow 0H & \swarrow 1H & \swarrow 2H \\ \downarrow & & \\ \text{TTT} & & \end{matrix}$

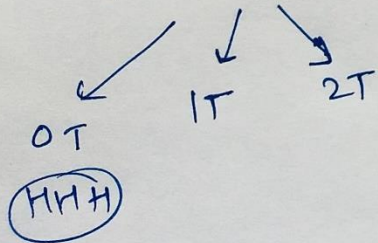
$$\textcircled{\text{V}} \quad P(\text{no head}) = \frac{1 \leftarrow \text{TTT}}{8}$$

$$\textcircled{\text{VI}} \quad P(3T) = \frac{1 \leftarrow \text{TTT}}{8}$$

$$\textcircled{\text{VII}} \quad P(\text{exactly } 2T) = \frac{3 \leftarrow \begin{array}{l} \text{TTH} \\ \text{HTT} \\ \text{THT} \end{array}}{8}$$

$$\textcircled{\text{VIII}} \quad P(\text{No tail}) = \frac{1 \leftarrow \text{HHH}}{8}$$

$$\textcircled{\text{IX}} \quad P(\text{atmost } 2T) = \frac{1+3+3}{8} = \frac{7}{8}$$



Q.9

Event 'A'

$$P(A) = \frac{2}{11} \quad \text{total}$$

$$P(\text{not } A) = P(A') = 1 - \frac{2}{11} = \frac{9}{11}$$

(Complementary event)

Q.10

ASSASSINATION

Vowels $\rightarrow \{A, A, A, I, I, O\} \rightarrow 6$

Consonants $\rightarrow \{S, S, S, S, N, T, N\} \rightarrow 7$

total = 13

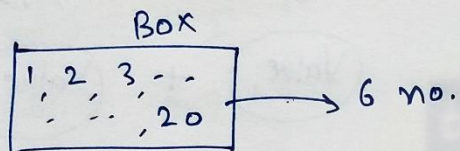
$$P(\text{vowel}) = \frac{6}{13} \quad \checkmark$$

$$P(\text{consonant}) = \frac{7}{13} \quad \checkmark$$

Toppers Village Class-11-Maths

Exercise-16.3 Probability

Q.11



"Committee has special
6 numbers" ← Prize

Person → 2, 7, 8, 10, 13, 15 *

Total No. of ways to select

6 numbers out of 20 = ${}^{20}C_6$

$$= \frac{20!}{14! \times 6!} = \frac{\cancel{20} \times 19 \times \cancel{18} \times 17 \times \cancel{16} \times 15 \times \cancel{14}!}{\cancel{14}! \times \underset{1}{\cancel{6}} \times \underset{1}{\cancel{5}} \times \underset{1}{\cancel{4}} \times \underset{1}{\cancel{3}} \times \underset{1}{\cancel{2}} \times \underset{1}{\cancel{1}}}$$

$$= 19 \times 17 \times 8 \times 15$$

$$= 38760$$

To win the prize money
a person has one way
to select only those
Special numbers.

Favourable cases = 1

$$P = \frac{1 \leftarrow \text{win}}{38760 \leftarrow \text{total}}$$

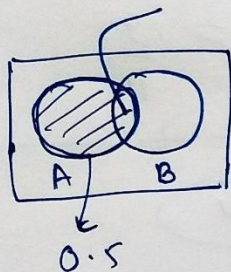
Q.12

Consistently defined $\begin{cases} \rightarrow \text{Yes?} \\ \rightarrow \text{No?} \end{cases}$

(i) $P(A) = 0.5$

$P(B) = 0.7$

$P(A \cap B) = 0.6 \leftarrow \text{common.}$



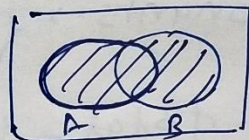
$P(A) < P(A \cap B)$

$\therefore$ Not Valid.

(ii) $P(A) = 0.5$

$P(B) = 0.4$

$P(A \cup B) = 0.8$



(Yes)

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cap B) = P(A) + P(B) - P(A \cup B)$
 $= 0.5 + 0.4 - 0.8 = 0.1$

Q.13

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) + P(A \cap B) = P(A) + P(B)$

	$P(A)$	$P(B)$	$P(A \cap B)$	$P(A \cup B)$
(i)	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{15}$	$\frac{7}{15}$
(ii)	0.35	0.5	0.25	0.6
(iii)	0.5	0.35	0.25	0.6

$\frac{1}{3} + \frac{1}{5} = \frac{1}{15} + x$
 $\frac{8}{15} = \frac{1}{15} + x$

Q.14 $P(A) = \frac{3}{5}$, $P(B) = \frac{1}{5}$

$P(A \text{ or } B)$ Mutually
Exclusive
events.

$P(A \cap B) = 0$

$P(A \cup B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{3}{5} + \frac{1}{5}$

$= \frac{4}{5}$

Q.15 $P(E) = \frac{1}{4}$

$P(F) = \frac{1}{2}$

$P(E \text{ and } F) = \frac{1}{8} = P(E \cap F)$

(i) $P(E \text{ or } F) = P(E \cup F)$
 $= P(E) + P(F) - P(E \cap F)$
 $= \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{2+4-1}{8}$
 $P(E \cup F) = \frac{5}{8}$

(ii) $P(\text{not } E \text{ and not } F)$
 $= P(E' \cap F')$

$= P[(E \cup F)']$

De Morgan's Law

$= 1 - P(E \cup F)$

$= 1 - \frac{5}{8} = \frac{3}{8}$ ✓

Q.15 $P(A) = 0.42$

$P(B) = 0.48$

$P(A \text{ and } B) = 0.16 = P(A \cap B)$

(i) $P(\text{not } A) = P(A')$
 $= 1 - P(A)$
 $= 1 - 0.42 = 0.58$

(ii) $P(\text{not } B) = P(B')$
 $= 1 - P(B)$
 $= 1 - 0.48 = 0.52$

(iii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
 $P(A \text{ or } B)$
 $= 0.42 + 0.48 - 0.16$
 $= 0.90 - 0.16$
 $= 0.74 \checkmark$

Q.16 $P(\text{not } E \text{ or not } F) = 0.25$

Mutually
exclusive
Events.

Yes? $P(E \cap F) = 0$
 No? $P(E \cap F) \neq 0$

$P(E' \cup F') = 0.25$

$\Rightarrow P(E \cap F)' = 0.25$

$\Rightarrow 1 - P(\underline{E \cap F}) = 0.25$

$\Rightarrow 1 - 0.25 = P(E \cap F)$

$\Rightarrow \underline{P(E \cap F) = 0.75 \neq 0}$

Not M.E.E.

Toppers Village class-11-Maths

Exercise-16.3 Probability.

Q.18

40% $\rightarrow$ Maths (M) $\rightarrow P(M) = 0.4$

30% $\rightarrow$ Bio (B) $\rightarrow P(B) = 0.3$

10% $\rightarrow$ Both $(B \cap M) \rightarrow P(B \cap M) = 0.1$

$$P(M \text{ or } B) = P(M \cup B) = P(M) + P(B) - P(M \cap B)$$

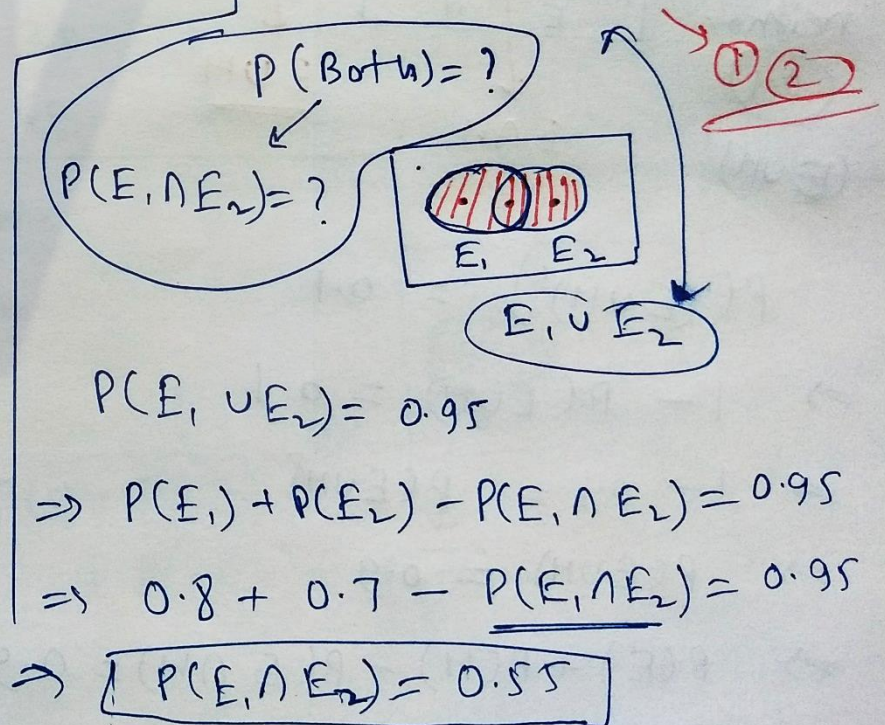
$$P(M \cup B) = 0.4 + 0.3 - 0.1 = 0.6$$

Q.19

$$P(E_1) = 0.8$$

$$P(E_2) = 0.7$$

कम से कम
 $P(\text{Passing at least one of them}) = 0.95$



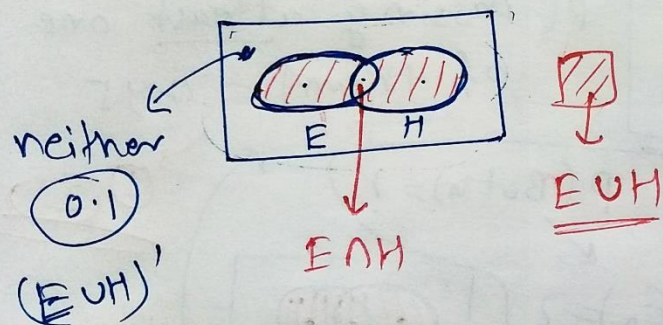
Q. 20

$$P(E \cap H) = 0.5$$

$$P(\text{neither}) = 0.1 = P((E \cup H)')$$

$$P(E) = 0.75$$

$$P(H) = ?$$



$$P((E \cup H)') = 0.1$$

$$\Rightarrow 1 - P(E \cup H) = 0.1$$

$$\Rightarrow 1 - 0.1 = P(E \cup H)$$

$$\Rightarrow P(E \cup H) = 0.9$$

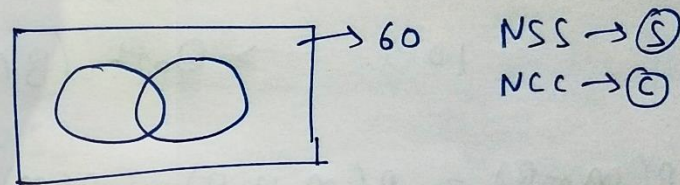
$$\Rightarrow P(E) + P(H) - P(E \cap H) = 0.9$$

$$\Rightarrow 0.75 + \underline{P(H)} - 0.5 = 0.9$$

$$\Rightarrow 0.25 + \underline{P(H)} = 0.9$$

$$\Rightarrow \boxed{P(H) = 0.65}$$

(21)



$$\text{total students} = 60 = n(T)$$

$$n(S) = 32, n(C) = 30$$

$$n(S \cap C) = 24$$

$$P(S) = \frac{n(S)}{n(T)} = \frac{32}{60} \checkmark$$

$$P(C) = \frac{n(C)}{n(T)} = \frac{30}{60} \checkmark$$

$$P(S \cap C) = \frac{n(S \cap C)}{n(T)} = \frac{24}{60} \checkmark$$

(i) Prob. of NCC or NSS

$$P(C \cup S) = P(C) + P(S) - P(C \cap S)$$

$$= \frac{30}{60} + \frac{32}{60} - \frac{24}{60} = \frac{38}{60}$$

(ii) Prob. of neither NCC nor NSS

$$P(C' \cap S') = P((C \cup S)')$$



$$P((C \cup S)') = 1 - P(C \cup S)$$

$$= 1 - \frac{38}{60} = \frac{22}{60}$$

(iii) Prob of NSS but not NCC.

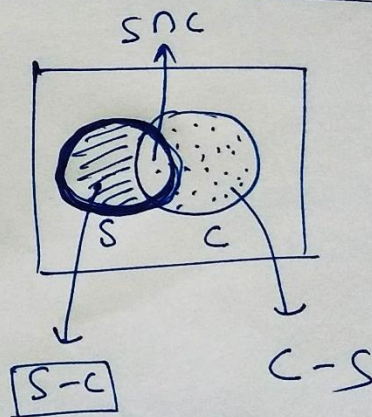
$$P(S \cap C') = P(S - C)$$

$$= P(S) - P(S \cap C)$$

$$= \frac{32}{60} - \frac{24}{60}$$

$$\rightarrow = \frac{8}{60}$$

$$P(S) = P(S - C) + P(S \cap C)$$

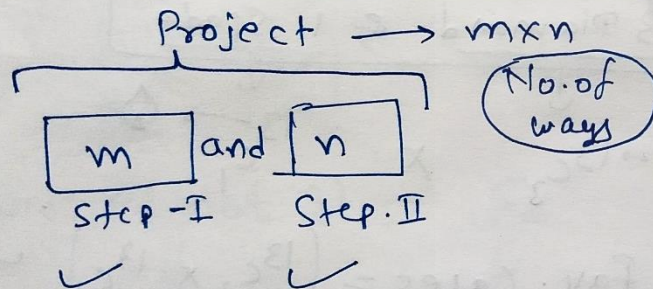


Miscellaneous Exercise - ch (16) Probability

• ${}^nC_r = \left(\begin{array}{l} \text{no. of ways to select} \\ r \text{ objects out of} \\ n \text{ - different objects.} \end{array} \right)$

$$\frac{n!}{(n-r)!r!} =$$

• Multiplication Rule of Counting



$$P = \frac{\text{fav.}}{\text{total}}$$

Q.1

10 red
20 Blue
30 Green

$\rightarrow$ 5 Balls

Total no. of cases = ${}^{60}C_5$ $\leftarrow$ total

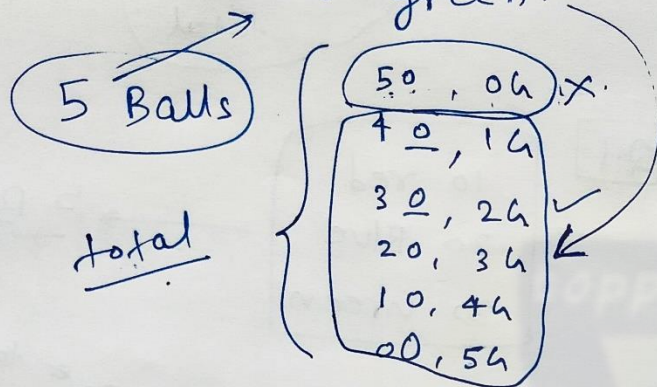
(i) all are blue.

5 $\rightarrow$ Blue $\leftarrow$ total Blue 20

fav. cases = ${}^{20}C_5$

$$P(\text{all blue}) = \frac{{}^{20}C_5}{{}^{60}C_5}$$

(ii) atleast one green.



$P(\text{atleast one green})$

$$= 1 - P(\text{no green})$$

total prob.

0 Green

from Red & Blue

$$= 1 - \frac{{}^{30}C_5}{{}^{60}C_5}$$

total

Q.2

52
Playing
Card

→ 4 cards

3 Diamond & 1 Spade

Total no. of ways = ${}^{52}C_4$ ✓

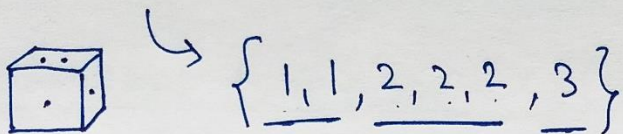
3 Diamond & 1 Spade

total → ${}^{13}C_3$ X ${}^{13}C_1$ → ♠

Fav. Cases = $({}^{13}C_3 \times {}^{13}C_1)$ ✓

$$P(3D \& 1S) = \frac{{}^{13}C_3 \times {}^{13}C_1}{{}^{52}C_4}$$

Q.3 Die (पारना)



$$(i) P(2) = \frac{3}{6} \begin{matrix} \leftarrow \text{fav. (2)} \\ \leftarrow \text{total} \end{matrix}$$

$$= \frac{1}{2}$$

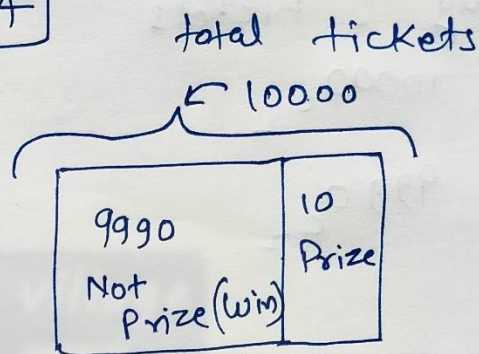
$$(ii) P(1 \text{ or } 3) = \frac{3}{6} = \frac{1}{2}$$

$\downarrow \qquad \qquad \downarrow$
 $\{1, 1\}$ $\{3\}$

$$(iii) P(\text{not } 3) = \frac{5}{6}$$

$\downarrow$
 $\{1, 1, 2, 2, 2\}$

Q.4



$$n_{C_r} = \frac{n!}{(n-r)! \cdot r!}$$

$$n_{C_1} = n$$

$P(\text{not winning}) = ?$

(a) if we buy 1 ticket

Total = $10000 C_1 = 10000$

favourable = $9990 C_1 = 9990$
 (not Prize)

$$P = \frac{9990}{10000} = \frac{999}{1000}$$

① if we buy 2 tickets

$$\text{total} = 10000c_2$$

$$\underline{\text{fav.}} = 9990c_2$$

$$P(\underline{\text{not winning}}) = \frac{9990c_2}{10000c_2} \checkmark$$

② if we ~~buy~~ buy 10 tickets.

$$\text{total} = 10000c_{10}$$

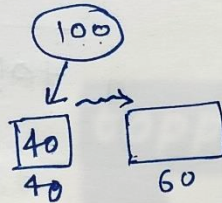
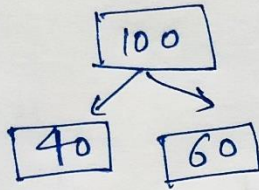
$$\text{fav.} = 9990c_{10}$$

$$P(\underline{\text{not winning}}) = \frac{9990c_{10}}{10000c_{10}} \checkmark$$

Toppers Village class-11-Maths

Miscellaneous Exercise - ch (16) Probability

Q.5

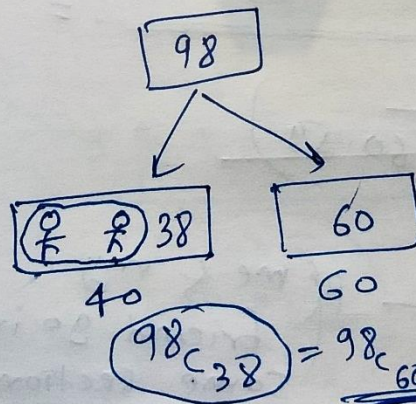
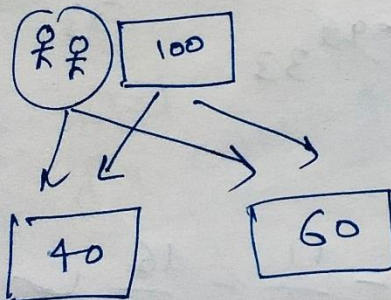


$$\text{total} = {}^{100}C_{40} \times {}^{100}C_{60} = \frac{100!}{40!60!}$$

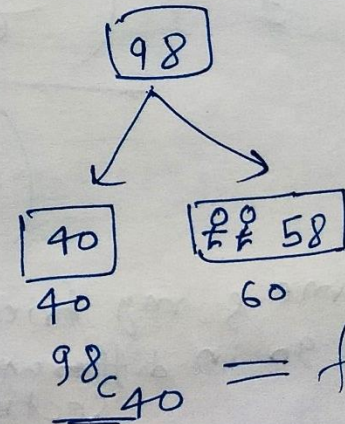
(selection)

Property $nCr = nCn-r$

(i) me & my friend are in same section.



or
+



$${}^{98}C_{40} = \text{fav. cases}$$

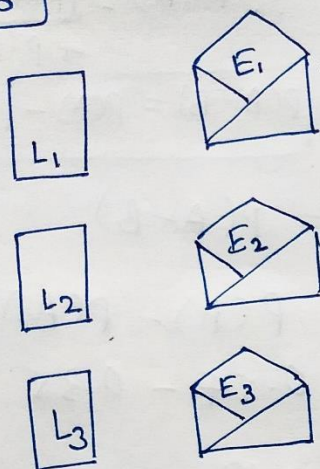
$P(\text{me \& my friend are in same section}) = \frac{{}^{98}C_{38} + {}^{98}C_{40}}{{}^{100}C_{40}} = \frac{\left[\frac{98!}{38! 60!} \right] + \left[\frac{98!}{40! 58!} \right]}{\left[\frac{100!}{40! 60!} \right]}$

$$= \frac{\left[\frac{98!}{38! 60 \cdot 59 \cdot 58!} \right] + \left[\frac{98!}{40 \cdot 39 \cdot 38! 58!} \right]}{\left[\frac{100 \times 99 \times 98!}{40 \cdot 39 \cdot 38! 60 \cdot 59 \cdot 58!} \right]} = \frac{\cancel{98!} \left[\frac{1}{60 \cdot 59} + \frac{1}{40 \cdot 39} \right]}{\cancel{98!} \left[\frac{100 \times 99}{40 \cdot 39 \cdot 60 \cdot 59} \right]}$$

$$= \frac{\left(\frac{40 \cdot 39 + 60 \cdot 59}{\cancel{60 \cdot 59 \cdot 40 \cdot 39}} \right)}{\left(\frac{100 \times 99}{\cancel{40 \cdot 39 \cdot 60 \cdot 59}} \right)} = \frac{\frac{17}{5100}}{\frac{100 \times 99}{33}} = \frac{17}{33} \checkmark$$

(b) $P(\text{me \& my friend go in different section}) = 1 - P(\text{me \& my friend go in same section}) = 1 - \frac{17}{33} = \frac{16}{33} \checkmark$

Q.6



1 letter $\rightarrow$ 1 envelope

Total = $3 \times 2 \times 1 = 6$
randomly fill

Answer.

$$P = \frac{4}{6} \leftarrow \text{fav.}$$

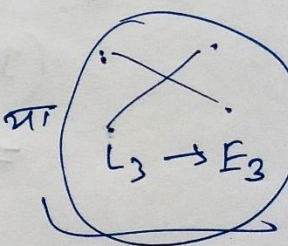
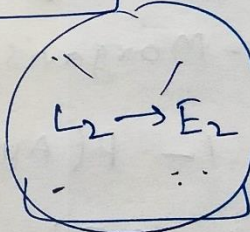
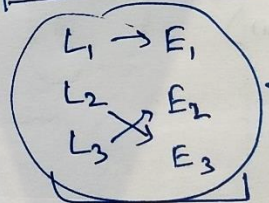
$\leftarrow$ total

$$= \frac{2}{3} \checkmark$$

Favourable Cases.

at least one letter into proper envelope
कम से कम

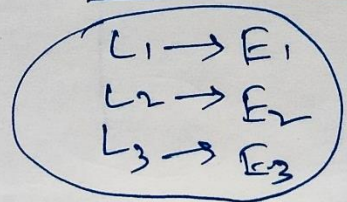
1 letter $\rightarrow$ Proper Envelope



3 Cases.

$\therefore$ total fav. cases = 4

2 letters $\rightarrow$ Proper Envelope



3 letter

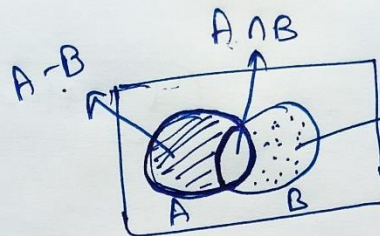
1-case

Q.7

$$P(A) = 0.54$$

$$P(B) = 0.69$$

$$P(A \cap B) = 0.35$$



$$\therefore P(A) = \boxed{P(A-B) + P(A \cap B)}$$

$$\Rightarrow \boxed{P(A-B) = P(A) - P(A \cap B)}$$

$$\begin{aligned} \text{(i)} \quad P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= 0.54 + 0.69 - 0.35 \\ &= 0.54 + 0.34 = 0.88 \quad \checkmark \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad P[A' \cap B'] &= P[(A \cup B)'] \\ &\quad \text{(De-Morgan's Law)} \\ &= 1 - P(A \cup B) \\ &= 1 - 0.88 \\ &= 0.12 \quad \checkmark \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad P(A \cap B') &= P(A - B) \\ &= P(A) - P(A \cap B) \\ &= 0.54 - 0.35 \\ &= 0.19 \quad \checkmark \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad P(B \cap A') &= P(B - A) \\ &= P(B) - P(A \cap B) \\ &= 0.69 - 0.35 \\ &= 0.34 \quad \checkmark \end{aligned}$$

Q.8

either - or -

$$P(\underbrace{\text{male}}_{(A)} \cup \underbrace{\text{over 35 years}}_{(B)}) = P(\text{male}) + P(\text{over 35 years}) - P(\text{male} \cap \text{over 35 years})$$

$$P(\text{male}) = \frac{3}{5} \leftarrow \begin{array}{l} \text{male} \\ \text{total} \end{array}$$

$$P(\text{over 35 years}) = \frac{2}{5}$$

$$P(\text{male} \cap \text{over 35 years}) = \frac{1}{5}$$

↓
(both)

put

$$= \frac{3}{5} + \frac{2}{5} - \frac{1}{5}$$
$$= \frac{4}{5}$$

Toppers Village Class-11-Maths

Miscellaneous Exercise - (16) probability

[Q. 9] (i) Digits are repeated

4 digit > 5000

0/1/3/5/7

total

5

(5000
div)

No. of ways

$$5 \cdot 5 \cdot 5 = 5 \cdot 5 \cdot 5 - 1$$

$$= 125 - 1$$

$$= 124$$

0/1/3/5/7

7

No. of ways

$$5 \cdot 5 \cdot 5 = 5 \cdot 5 \cdot 5$$

$$= 125$$

$$\text{total} = 249$$

$$P(\text{No. divisible by } 5) = \frac{\text{fav.}}{\text{total}} = \frac{99}{249} = \frac{33}{83}$$

fav.
Cases

No.
divisible
by
5

0/5
last Digit

0/1/3/5/7

0/5

5

5

5

2

$$= 5 \cdot 5 \cdot 2 - 1$$

$$= 49$$

7

5

5

2

$$= 50$$

$$\text{total fav.} = 99$$

(5000
div)

Q.9 (ii) The repetition of digits is not allowed.

4 digit > 5000

0 | 1 | 3 | 5 | 7

0 | 1 | 3 | 7

total.

No. of ways → $\overset{5}{-} \overset{-}{-} \overset{-}{-} \overset{-}{-} = 24$

0 | 1 | 3 | 5

7

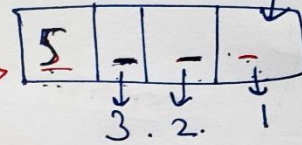
No. of ways → $\overset{-}{-} \overset{-}{-} \overset{-}{-} \overset{-}{-} = 24$

total = 48

$$P(\text{No. is divisible by 5}) = \frac{\text{fav.}}{\text{total.}} = \frac{18}{48} = \frac{3}{8}$$

Repetition is not allowed

1 | 3 | 7



= 6



= 12

1 | 3 | Remaining

total = 18
fav.

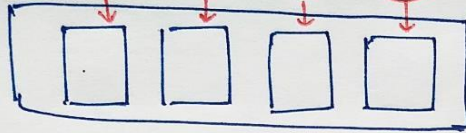
Fav. cases

No. is divisible by 5

last Digit 0/5

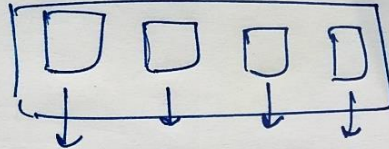
Q.10

Lock →



No repeats

Total =



10 x 9 x 8 x 7

0/1/2/.../9

$$\begin{aligned}\text{total} &= 10 \times 9 \times 8 \times 7 \\ &= 10 \times 9 \times 56 \\ &= 5040\end{aligned}$$

$$P(\text{get the right sequence to open the lock}) = \frac{1}{5040}$$

↓ fav.
total

Note: To open a lock, we always have only one code (password).

∴ No. of favourable cases = 1

The End

All Useful Links:

- **YouTube Channel : Toppers Village** (Get full and free video lectures of Class 10 & 11 Maths) - <https://www.youtube.com/toppersvillage>
- **Website :** www.toppersvillage.com
- **Instagram :** <https://www.instagram.com/toppersvillage/>
- **Facebook Page:** www.facebook.com/ToppersVillage/
- **Facebook Group:** <https://www.facebook.com/groups/ToppersVillage>